Homework 2. Linear Algebra. Spring 2022. Prof. Pineiro

Print Name: _____

1. For the matrix
$$A = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$$
   (a) Find the determinant of A.
   (b) Is A invertible? Explain your answer.
   (c) Find the adjoint matrix $\text{adj}(A)$.
   (d) Find the inverse if possible.

2. For the matrix
$$A = \begin{pmatrix} 1 & 1 & 3 \\ 3 & 1 & -5 \\ 2 & 1 & -1 \end{pmatrix}$$
   (a) Find the determinant of A.
   (b) Is A invertible? Explain your answer.
   (c) Find the adjoint matrix $\text{adj}(A)$.
   (d) Find the inverse if possible.

3. For the matrix:
$$A = \begin{pmatrix} 1 & 1 & 1 & 4 \\ 3 & 1 & 0 & 9 \\ 1 & 1 & 2 & 3 \\ 1 & -1 & -2 & 3 \end{pmatrix}$$
   (a) Find the determinant $\det(A)$. Hint: Multiply the first column by -3 and add the result to the last column and compute the determinant of the new matrix instead.
   (b) Is the matrix A invertible? Explain your answer.
   (c) Using the fact that the adjoint matrix of A is given by $\text{adj}(A) = \begin{pmatrix} 12 & -6 & -2 & 4 \\ 0 & -4 & 6 & 6 \\ 0 & 2 & -4 & -2 \\ -4 & 2 & 0 & -2 \end{pmatrix}$.
   Find the inverse A^{-1}.

(d) Use your result to solve the system:

$$\begin{cases} x_1 + x_2 + x_3 + 4x_4 = 1 \\ 3x_1 + x_2 + 9x_4 = 1 \\ x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ x_1 - x_2 - 2x_3 + 3x_4 = 1 \end{cases}$$

4. Given the matrices:

$$A = \begin{pmatrix} 4 & 2 & 2 \\ -1 & -3 & -1 \\ -1 & 1 & -2 \end{pmatrix} \qquad B = \begin{pmatrix} \frac{7}{18} & \frac{1}{3} & \frac{2}{9} \\ -\frac{1}{18} & -\frac{1}{3} & \frac{1}{9} \\ -\frac{2}{9} & -\frac{1}{3} & -\frac{5}{9} \end{pmatrix}$$

Check that the two matrices are inverse to each other.